Hypothesis Testing for Proportions

Section 9.3
Lecture 30

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Outline

The Steps of Testing a Hypothesis
p-Value Approach

The seven steps:

1. State the null and alternative hypotheses.
2. State the significance level.
3. State the formula for the test statistic.
4. Compute the value of the test statistic.
5. Compute the p-value.
6. Make a decision.
7. State the conclusion.

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An Example

The following is a minimal write-up of the problem.

1. Let p = the proportion of male births.
$$H_0 : p = 0.50$$
$$H_1 : p > 0.50$$
2. $\alpha = 0.05$.
3. $Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$.
4. $Z = 1.265$.
5. p-value $= 0.1029$.
6. Do not reject H_0.
7. The proportion of male births is 0.50.

Male Births vs. Female Births

- Are male births more likely than female births?
  - More Boys Born Than Girls

Hypothesis Testing on the TI-83

The TI-83 has special functions designed for hypothesis testing.

- Press `STAT`.
- Select the `TESTS` menu.
- Select `1-PropZTest...`
- Press `ENTER`. A window appears.
- Enter the value of p_0.
- Press `ENTER` and the down arrow.
- Enter the numerator x of $\hat{p}$.
- Press `ENTER` and the down arrow.
- Enter the sample size n.
- Press `ENTER` and the down arrow.

Hypothesis Testing on the TI-83

The TI-83 has special functions designed for hypothesis testing.

- Select the type of alternative hypothesis.
- Press the down arrow.
- Select `Calculate`.
- Press `ENTER`.

Hypothesis Testing on the TI-83

The display shows

- The title `1-PropZTest`.
- The alternative hypothesis.
- The value of the test statistic Z.
- The p-value.
- The value of $\hat{p}$.
- The sample size n.

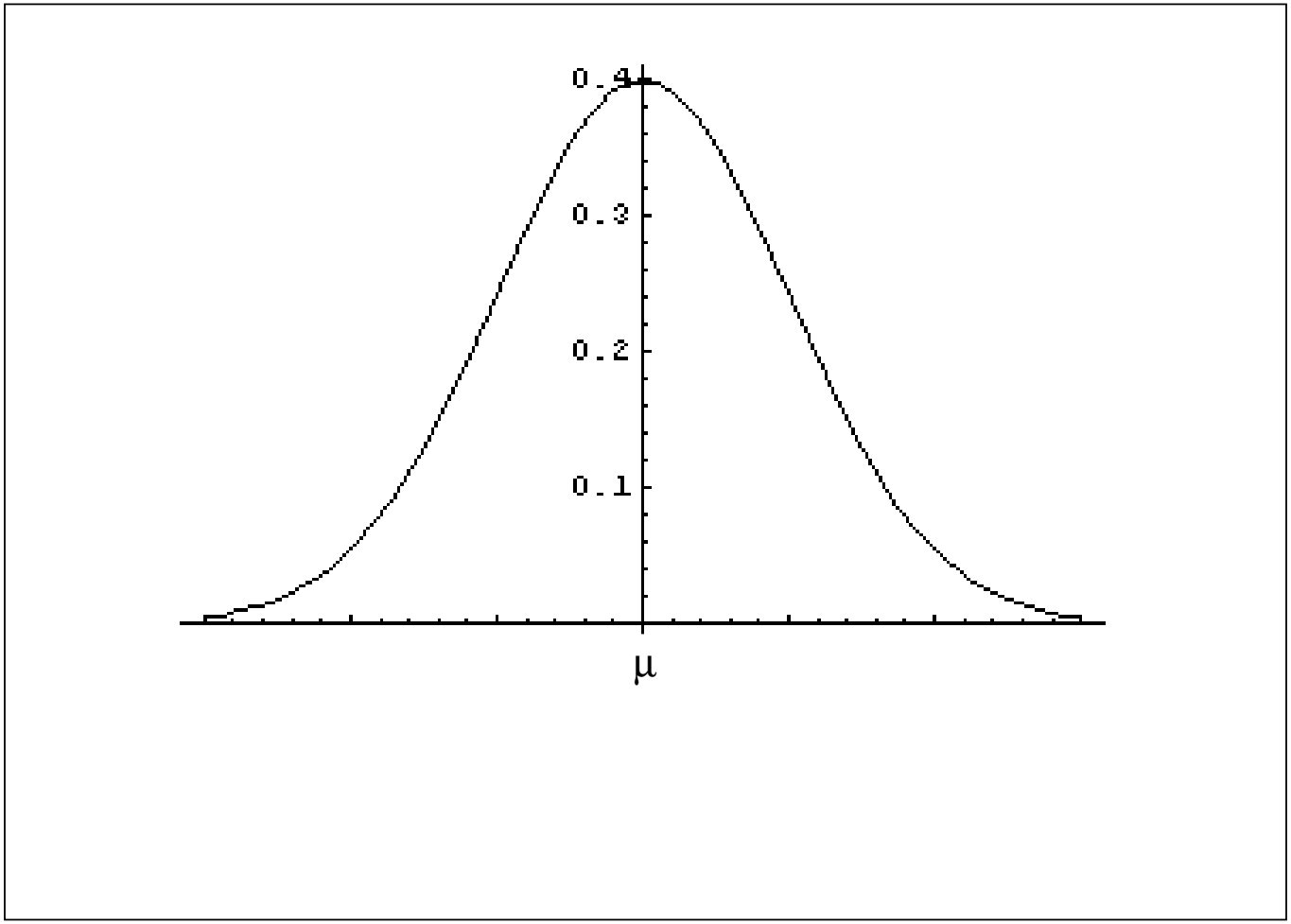
Example

- Studies have shown that moderate exercise helps reduce the risk of catching a cold.
  - Moderate Exercise May Lower Cold Risk
- Assume that 5 people caught colds out of 53 who did moderate exercise.
- Use the TI-83 to test the hypothesis that a person who gets moderate exercise has less than a 30% chance of catching a cold.

The Concept behind the Classical Approach

- Specify α.
- Determine the critical value and the rejection region.
- See whether the value of the statistic falls in the rejection region.
- Report the decision.

The Classical Approach



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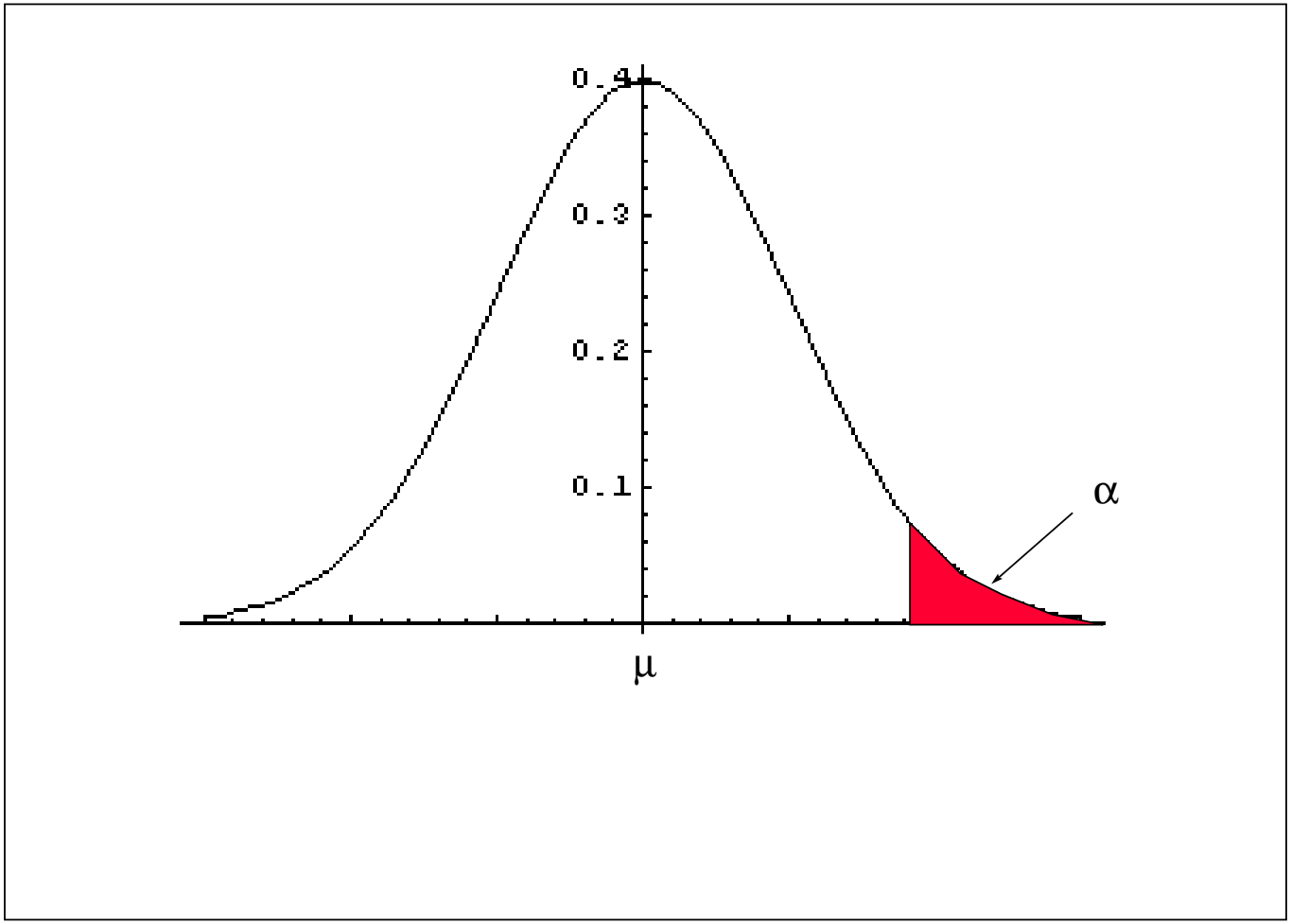

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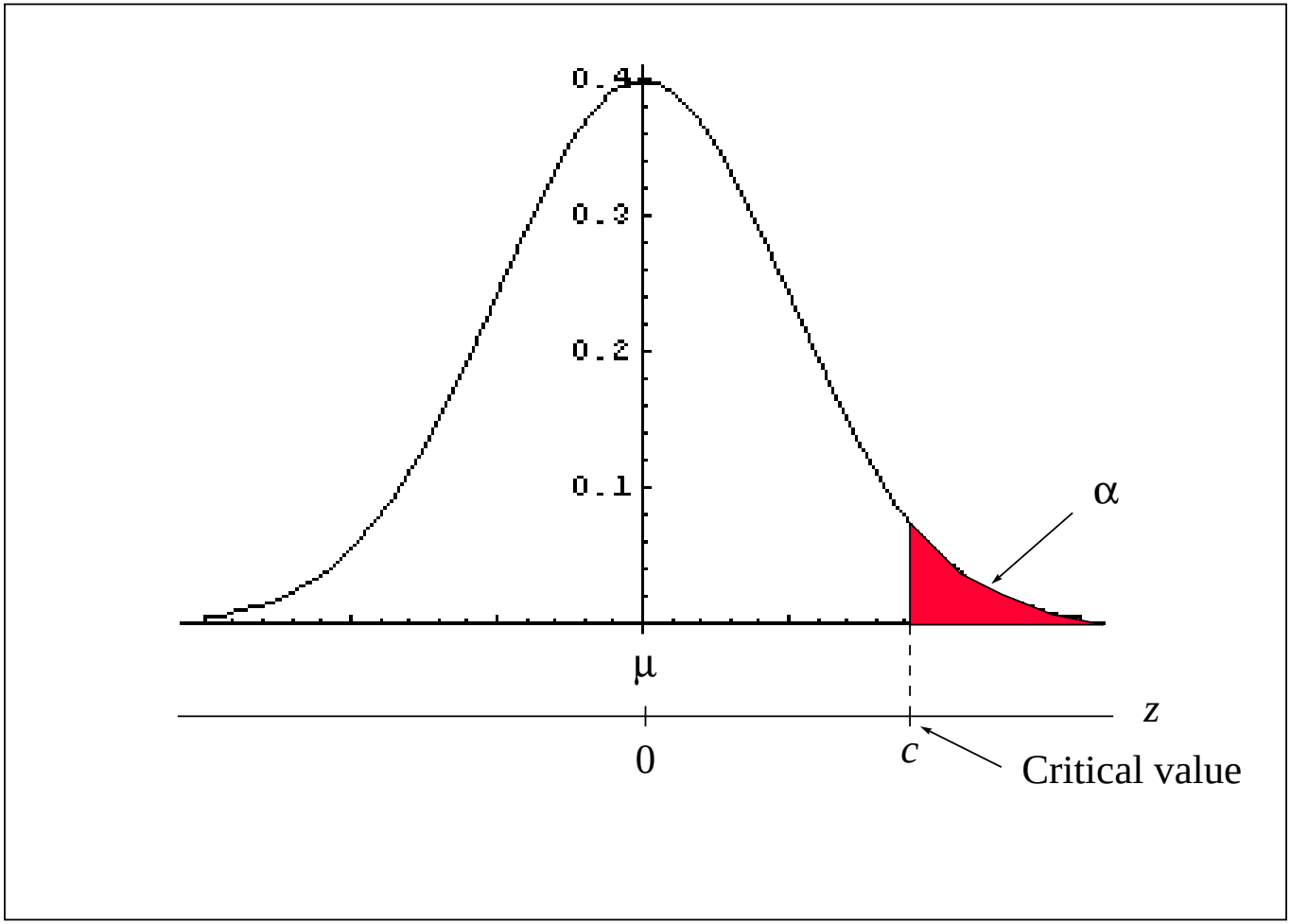

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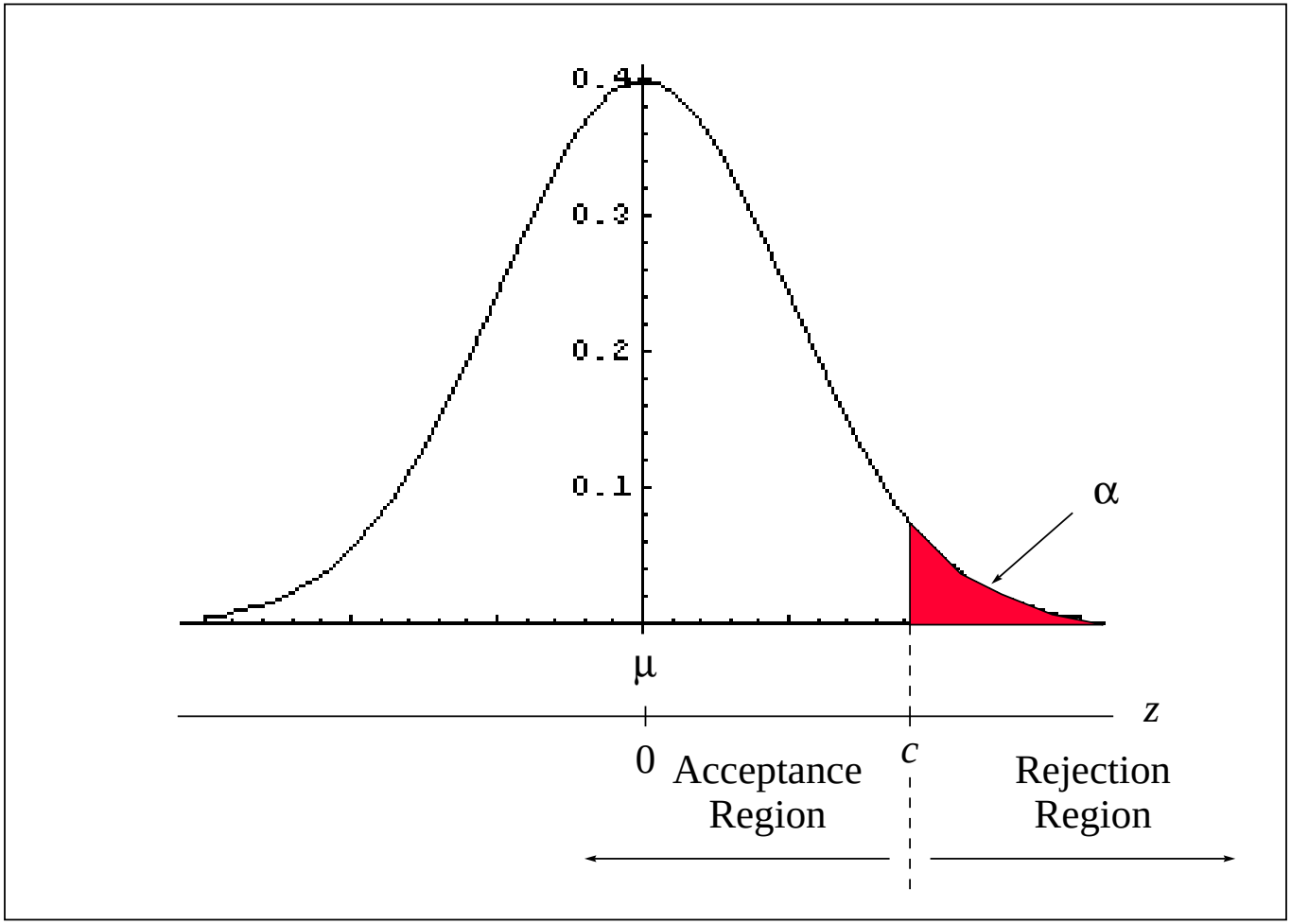

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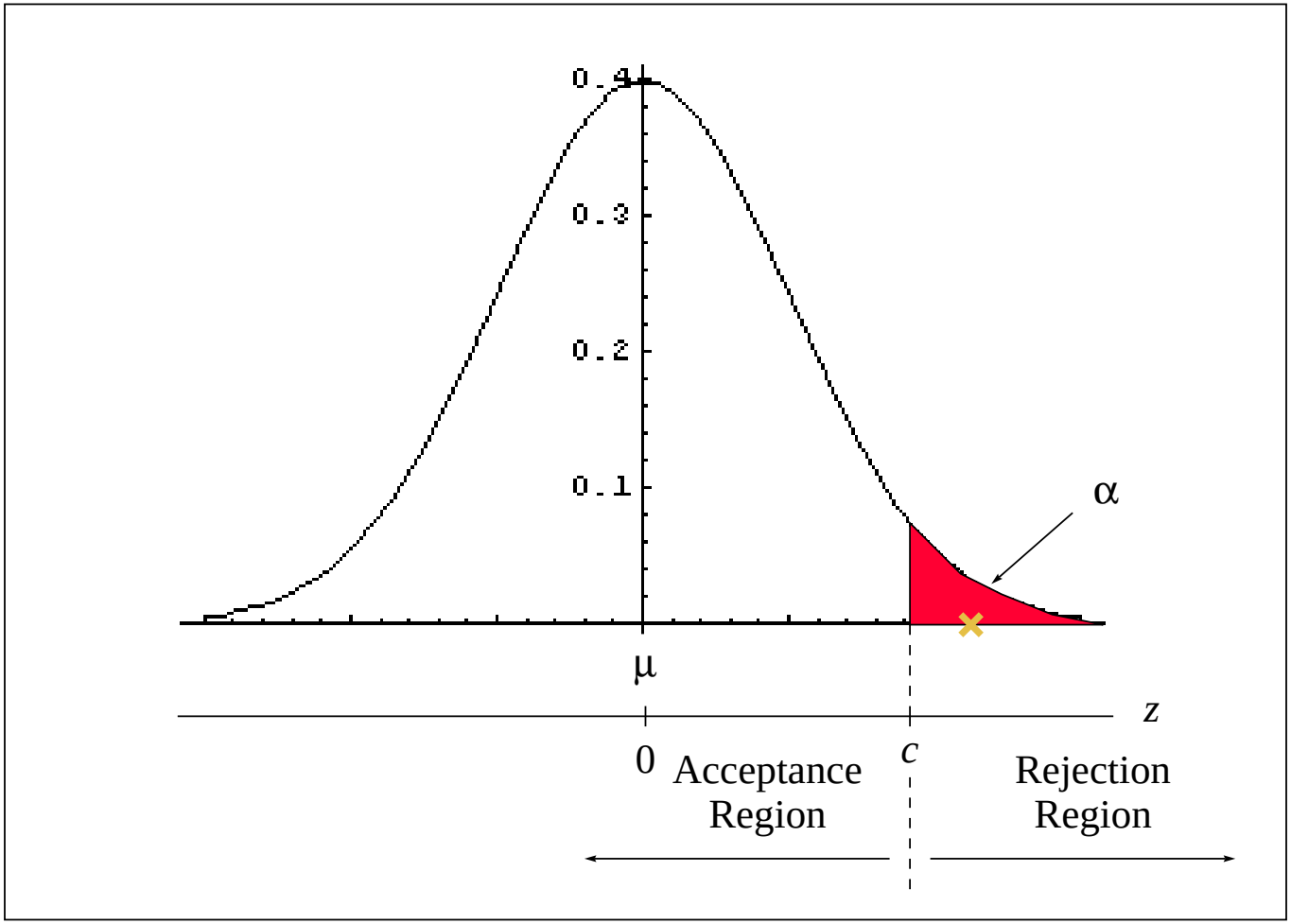

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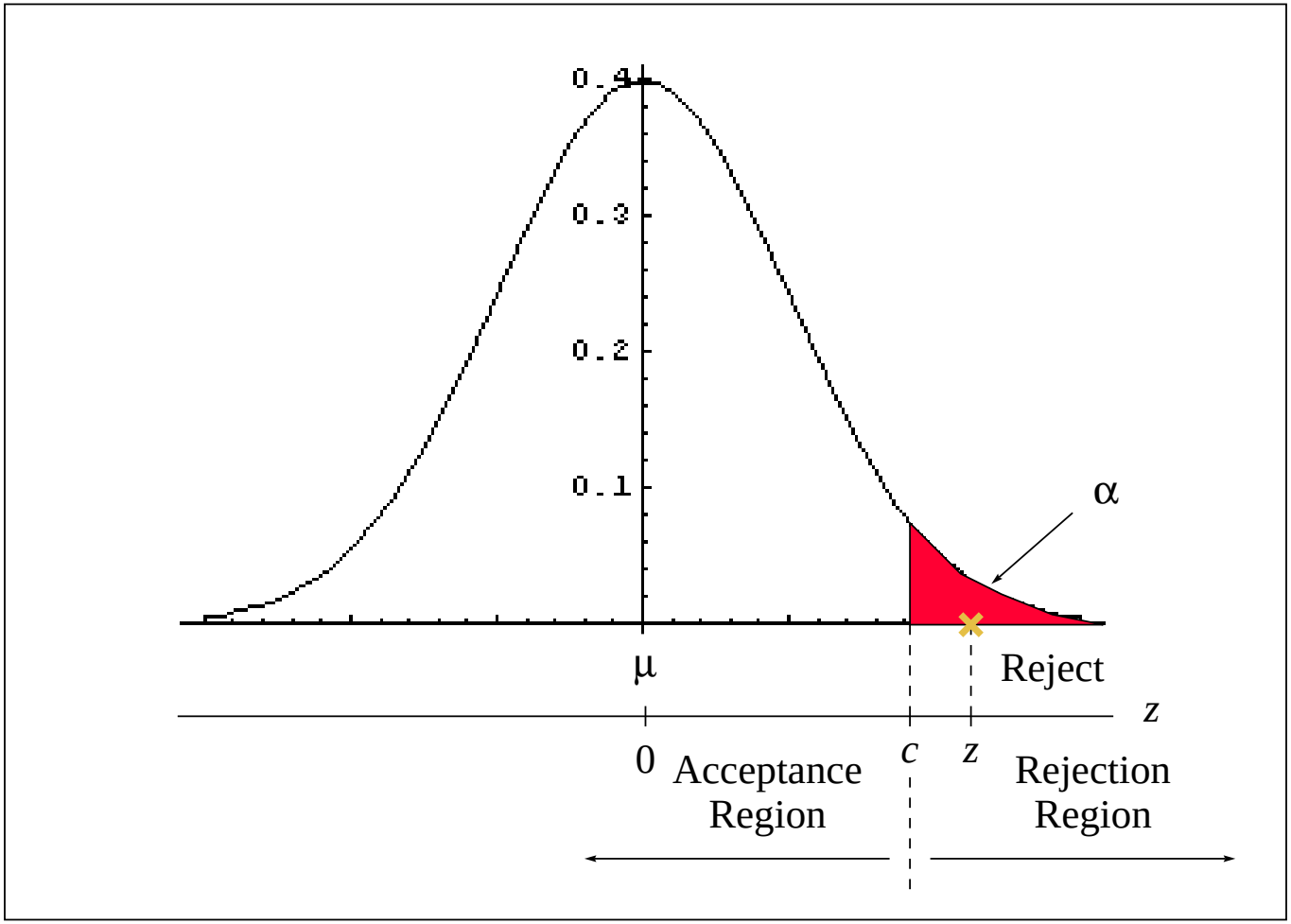

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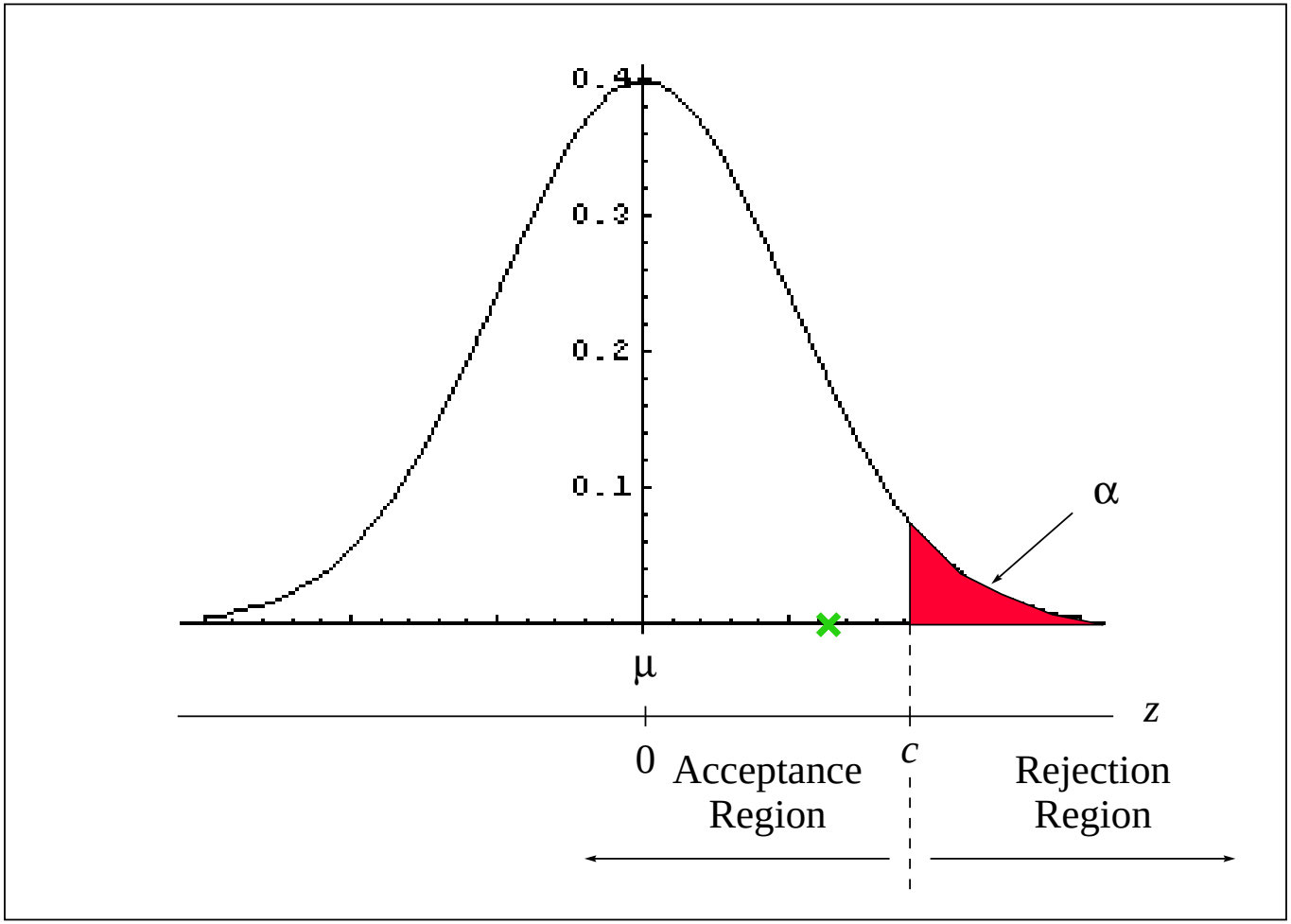

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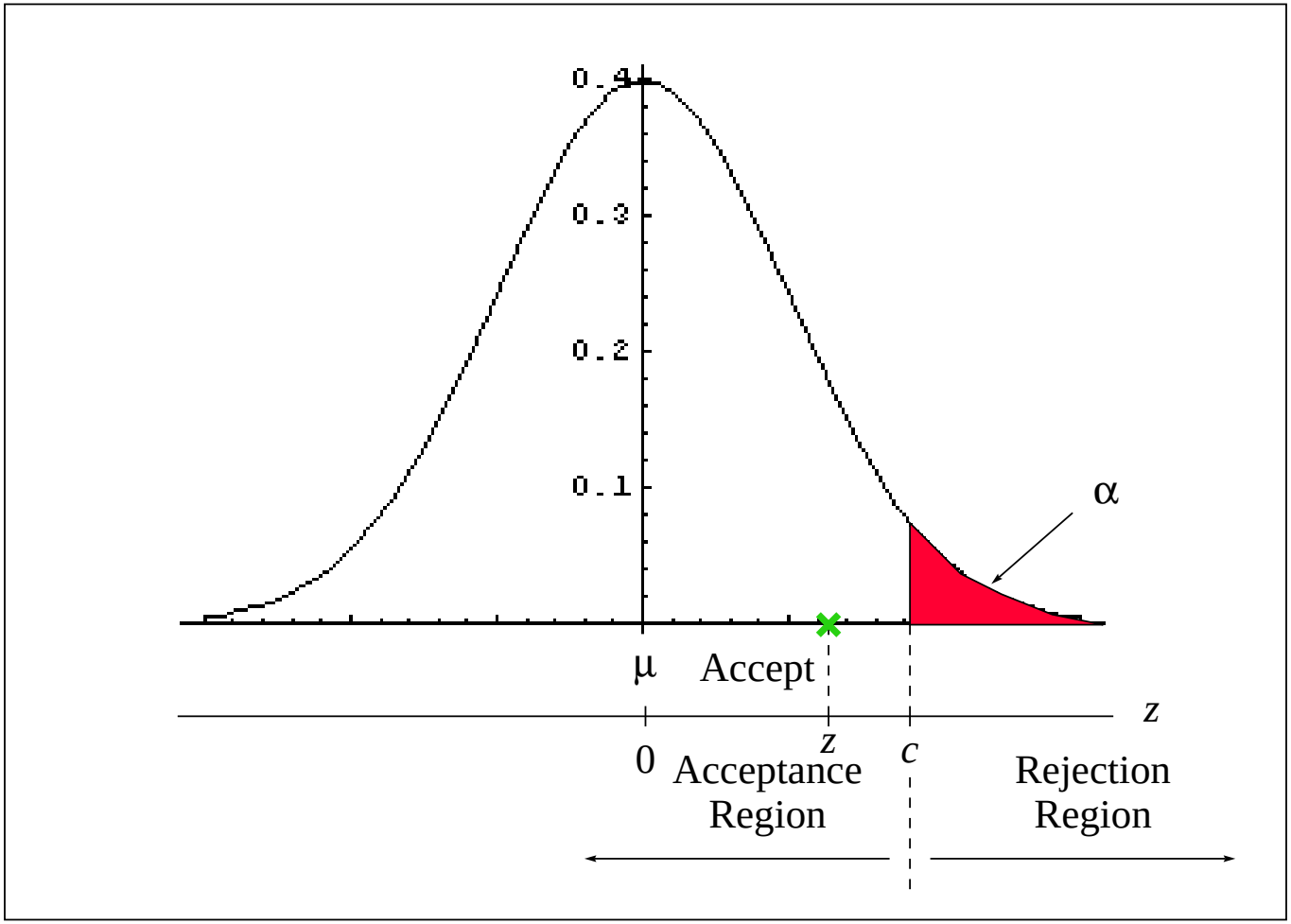

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Classical Approach

The seven steps:

1. State the null and alternative hypotheses.
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An Illustrative Example

We will use the following example to illustrate the seven steps.

- Are male births more common than female births?
- Suppose a random sample of 1000 live births shows that 520 are males and 480 are females.
- Test the hypothesis that male births are more common than female births, at the 5% level of significance.

The Hypotheses

- The null and alternative hypotheses are

$$H_0 : p = 0.50.$$
$$H_1 : p > 0.50.$$

The Level of Significance

- The significance level is

$$\alpha = 0.05$$

The Formula for the Test Statistic

- The formula for the test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

The Decision Rule

- We need to find out what value of z cuts off a tail of area α.
- Since $\alpha = 0.05$ and the direction of extreme is to the right, we have

$$z = \texttt{invNorm(0.95)} = 1.645.$$

- The decision rule is

Reject the null hypothesis if

$$Z > 1.645.$$

The Value of the Test Statistic

- The value of the test statistic is $\hat{p} = \frac{520}{1000} = 0.52$, $p_0 = 0.50$, and $n = 1000$.

$$\begin{aligned} Z &= \frac{0.52 - 0.50}{\sqrt{\frac{(0.50)(1-0.50)}{1000}}} \\ &= \frac{0.02}{0.01581} \\ &= 1.265 \end{aligned}$$

The Decision

- Compare the value of the test statistic to the decision rule.
- The value of z is not greater than 1.645, so our decision is:

  Do not reject the null hypothesis.

The Conclusion

- The conclusion is

  The proportion of male births is equal to 50%.

Summary

The following is a minimal write-up of the problem using the classical method.

1. Let p = the proportion of male births.
$$H_0 : p = 0.50$$
$$H_1 : p > 0.50$$
2. $\alpha = 0.05$.
3. $Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$.
4. Reject H_0 if $Z > 1.645$.
5. $z = 1.265$.
6. Do not reject H_0.
7. The proportion of male births is 0.50.